Number Properties

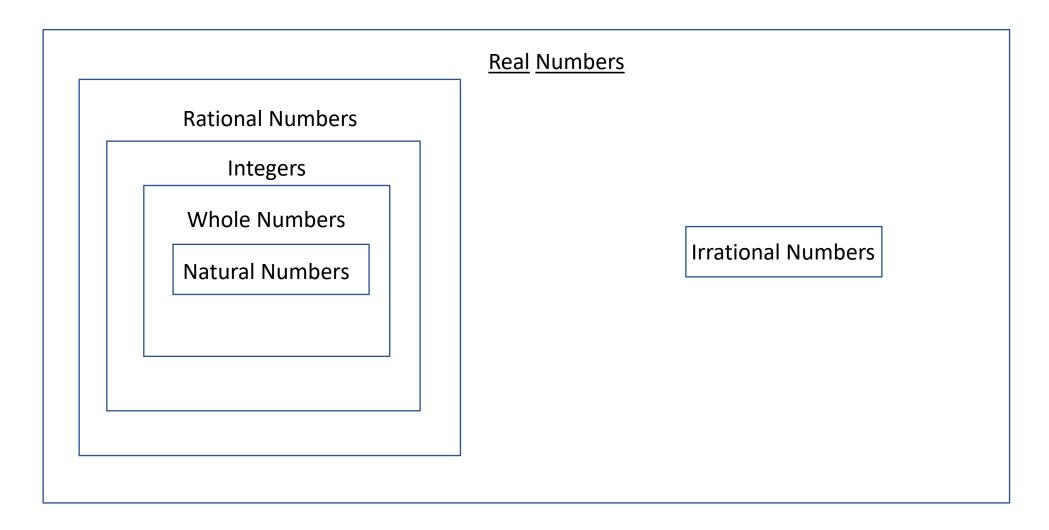# Types of Numbers

Real Numbers

Rational Numbers

Integers

Whole Numbers

Natural Numbers

Irrational Numbers

Types of Numbers

Natural Numbers – All +ve integers from 1 to ∞

Whole Numbers – All Natural Numbers from 0 to ∞

Integers – All +ve & -ve numbers from - ∞ to +∞

Rational Numbers – Numbers that can be expressed as a fraction of an integer & non-zero integer(denominator)

Irrational Numbers – A Real Number which is not rational like $\sqrt{5}$

Real Numbers – All numbers that can be expressed as a limit of a sequence of Rational Numbers

P.S. For the GMAT & GRE, Operations on Irrational Numbers are not tested, however an irrational number as an answer to a question is a common occurrence

Divisibility Rules

Divisibility Rules

2	4	8
The last digit of the number should be divisible by 2	The last TWO digits of the number should be divisible by 4	The last THREE digits of the number should be divisible by 8
Example	Example	Example
458 – The last digit of 458 is 8 8 is divisible by 2 Therefore, 458 is divisible by 2	6456 – The last TWO digits of 6456 are 56 56 is divisible by 4 (56÷4 = 14) Therefore, 6456 is divisible by 4	127896 – The last THREE digits of 127896 are 896 896 is divisible by 8 (896÷8 = 112) Therefore, 127896 is divisible by 8

Divisibility Rules

3

The sum of the digits of the number should be divisible by 3

Example

573 –
5+7+3=15
15 is divisible by 3
Therefore, 573 is divisible by 3

6

- The sum of the digits of the number should be divisible by 3
- The last digit of the number should be divisible by 2

Example

7968 –
7+9+6+8= 30
30 is divisible by 3
The last digit of 7968 is 8
8 is divisible by 2
Therefore, 7968 is divisible by 6

9

The sum of the digits of the number should be divisible by 9

Example

4104 –
4+1+0+4= 9
9 is divisible by 9
Therefore, 4104 is divisible by 9

Divisibility Rules

5	10	11
The last digit of the number should be 5 or 0	The last digit of the number should be 0	The difference between the sum of the digits in the odd <u>places</u> and the sum of the digits in the even <u>places</u> should be 0 or a multiple of 11
Example	Example	Example
955 – The last digit of 955 is 5 Therefore, 955 is divisible by 5	9320 – The last digit of 9320 is 0 Therefore, 9320 is divisible by 10	5016 – Sum of Odd <u>place</u> digits – (6+0) = 6 Sum of Even <u>place</u> digits – (1+5) = 6 Difference between sum of odd and even place digits – (6-6) = 0 Therefore, 5016 is divisible by 11

Factors & Multiples

Definitions

If a is a multiple of b then, we can write it mathematically as the following

$$a = kb, \quad \text{where k is any integer}$$

If a is a factor of b then, we can write it mathematically as the following

$$b = ka, \quad \text{where k is any integer}$$

How to determine the number of factors for any given number?

To find the number of factors of a given number, you need to first break the number into powers of its prime factors and add 1 to their powers & multiply them

Example:

$$72 = 2^3 \times 3^2$$

Number of factors = (3+1) x (2+1) = 12

How to determine the SUM of factors for any given number?

To find the sum of all the possible factors of a given integer n, where n = p^a x q^b x r^c , we use the following formula:

Sum of factors of integer n = $\dfrac{(p^{(a+1)}-1) \text{ X} (q^{(b+1)}-1) \text{X} (r^{(c+1)}-1)}{(p-1)\ (q-1)(r-1)}$

Example:

$1800 = 2^3$ x 3^2 x 5^2

Sum of all factors = $\dfrac{(2^{(3+1)}-1) \text{ X}(3^{(2+1)}-1)\text{X}(5^{(2+1)}-1)}{(2-1)\ (3-1)(5-1)} = \dfrac{(16-1)(27-1)(125-1)}{(1)(2)(4)} = \dfrac{15 * 26 * 124}{8} = 6045$

Note: Number of factors of 1800 = (3+1) x (2+1) x (2+1) = 36

Prime & Composite Numbers

Definitions

Prime numbers are numbers that have only TWO FACTORS
One factor is the number itself and the other is 1

Example:

       2, 3, 5, 7, 11, 13

These are the first few prime numbers on the number line

To Check if a number n is a prime number:

       Divide n by all the numbers from 2 to $\sqrt{n}$
       If n is divisible by at least one number in
       between then it is not a prime number

However, if none of those numbers can perfectly divide n, then n is a prime number

Definitions

Composite numbers are defined as any number that has more than 2 factors including 1 and itself

Example:

        4, 6, 8, 9, 10, 12

These are the first few composite numbers on the number line

To Check is a number n is a composite number:

        Divide n by all the numbers from 2 to $\sqrt{n}$
        If n is divisible by at least one number in
        between then it is a composite number

Properties of Prime & Composite Numbers

- 1 is neither prime nor composite

- Any number that is prime cannot be composite

- Any number that is composite cannot be prime

- All prime numbers greater than 3, can be expressed by the following expression:

$$6n-1 \text{ or } 6n+1$$

This expression is useful when trying to solve for or find a prime number
and hence we can represent the number as the above expression

HCF & LCM

Definitions

Least Common Multiple or LCM of two integers
is the smallest common multiple of those two integers

Calculating LCM:

The method used to calculate LCM is called Prime Factorization

Prime Factorization of 2 or more numbers is performed simultaneously for all numbers under one operation

Example:

LCM of 4, 6 & 8

$$
\begin{array}{c|c}
2 & 4,6,8 \\
2 & 2,3,4 \\
2 & 1,3,2 \\
3 & 1,3,1 \\
\hline
& 1,1,1
\end{array}
$$

2 x 2 x 2 x 3 = 24

Here we can see how Prime Factorization works.
Using prime factors that are common to all numbers and then subsequently using uncommon factors to reduce all the numbers to 1

Definitions

Greatest Common Divisor or Highest Common Factor of two integers
is the largest common factor/divisor of those two integers

Calculating GCD or HCF:

The method used to calculate GCD is also called Prime Factorization, however, unlike
LCM where the numbers are operated on together, in GCD, the numbers are operated on
separately

Example:

GCD of 4, 6

2	4		2	6
2	2		3	3
	1			1

Here, we can see that 2 is the
greatest common factor

Properties of LCM & GCD

- Both LCM & GCD use Prime Factorization to solve

- LCM solves for all numbers simultaneously

- GCD solves for each number separately

- For any two numbers a & b:

$$a \times b = \text{LCM of } (a,b) \times \text{GCD of } (a,b)$$

Even & Odd Numbers

Definitions

Even numbers are numbers that can be perfectly divided by 2 without leaving any remainder

Example:

2, 4, 6, 8, 10, 12, 14

These are the first few even numbers on the number line

Representing a number n as an even number:

$n = 2k$, where k is any integer

To represent consecutive even numbers we express as follows:

$(n-2), n, (n+2) = (2k-2), 2k, (2k+2)$, where n is the middle number

However, any sequence of numbers can be represented as multiples of 2 and an increasing and recurring difference of 2 with every consecutive number

Definitions

Odd numbers are numbers that cannot be perfectly divided by 2 without leaving any remainder, which means they leave 1 as a remainder when divided by 2

Example:

3, 5, 7, 9, 11, 13

These are the first few odd numbers on the number line

Representing a number n as an odd number:

$$n = 2k+1 \text{ , where k is any integer}$$

To represent consecutive odd numbers we express as follows:

$$(2k-3), (2k-1), (2k+1), (2k+3)$$

Properties of Even & Odd Numbers

Addition:

E + E = E
E + O = O
O + E = O
O + O = E

Subtraction:

E - E = E
E - O = O
O - E = O
O - O = E

Multiplication:

E x E = E
E x O = E
O x E = E
O x O = O

Division:

E ÷ E = E or O or Fraction
E ÷ O = E or O or Fraction
O ÷ E = Fraction
O ÷ O = E or O or Fraction

E represents EVEN NUMBERS
O represents ODD NUMBERS

Positive & Negative Numbers

Definitions

To understand Positive and Negative Numbers, it is important to understand what the Number Line is

Number Line:

Absolute
Zero

Positive Numbers

-∞   -6   -5   -4   -3   -2   -1   0   1   2   3   4   5   6   ∞

Negative Numbers

Properties of Positive & Negative Numbers

Addition:	Subtraction:	Multiplication:	Division:
P + P = P	P - P = P or N	P x P = P	P ÷ N = N
P + N = P or N	P – N = P	P x N = N	N ÷ P = N
N+ P = P or N	N – P = N	N x P = N	N ÷ N = P
N + N = N	N – N = P or N	N x N = P	P ÷ P = P

P represents POSITIVE NUMBERS
N represents NEGATIVE NUMBERS

Arithmetic Sequences

Definitions

A sequence of numbers such that the difference between consecutive terms is constant

Example:

a, a+d, a+2d, a+3d,….

Representing an arithmetic sequence:

T_n is the nth term of the sequence

T_n = a + (n-1)d , where a is the first term of the sequence and d is the common difference

Calculating the *number of terms* of an arithmetic sequence, *n*:

$n = \dfrac{T_n - T_1}{d} + 1$, where T_n is the last term of the sequence

and T_1 is the first term of the sequence
and d is the common difference

Contd...

Calculating the _sum_ of an arithmetic sequence, S_n: **(Method 1)**

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

Calculating the _sum_ of an arithmetic sequence, S_n: **(Method 2)**

$$S_n = n\ \frac{a+a+(n-1)d}{2}$$

Calculating the _sum_ of an arithmetic sequence, S_n: **(Method 3)**

$$S_n = [\text{number of terms}] \times [\text{mean of first term \& last term}]$$

Exponents & Roots

Definitions

Exponents

a quantity representing the power to which a given number or expression is to be raised, usually expressed as a raised symbol beside the number or expression

Example:

$$2^3 = 2 \times 2 \times 2 = 8$$

3 is the exponent

Roots

a number or quantity that when multiplied by itself, typically a specified number of times, gives a specified number or quantity.

Example:

$$\sqrt[3]{8} \text{ or } 8^{1/3} = 2 \times 2 \times 2$$

$\sqrt[3]{}$ is the root

OR

$1/3$ is the exponent

Properties of Exponents

$(-k)^n$ = +ve if n is even

    = -ve if n is odd

    = 1 if n is 0

0^n = 0 (always for ALL values of n)

1^n = 1 (always for ALL values of n)

$(-1)^n$ = -1 if n is odd

    = 1 if n is even

Properties of Exponents

$$\left(\frac{x}{y}\right)^a = \frac{x^a}{y^a}$$

$$x^a \times y^a = (xy)^a$$

$$(x^a)^b = x^{ab} = \left(x^b\right)^a$$

$$a^x \times a^y = a^{x+y}$$

$$a^x \div a^y = a^{x-y}$$

Properties of Exponents

$$a^{-x} = \frac{1}{a^x}$$

$$a^{\frac{x}{y}} = \sqrt[y]{a^x} = \left(\sqrt[y]{a}\right)^x$$

$$a^x + a^x + a^x = 3\,a^x$$

Properties of Exponents

$x^n - y^n$ is ALWAYS divisible by $(x - y)$

$x^n - y^n$ is divisible by $(x + y)$ if n is even

$x^n + y^n$ is divisible by $(x + y)$ ONLY if n is odd

Properties of Roots

$$\sqrt{a} \times \sqrt{b} = \sqrt{a}b$$

$$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$$

$$(\sqrt{a})^n = \sqrt{a^n}$$

$$a^{\frac{x}{y}} = \sqrt[y]{a^x} = \left(\sqrt[y]{a}\right)^x$$

$\sqrt{a^2}$ = |a| [on the GMAT & GRE all ROOTS are POSITIVE]

$\sqrt{a^2}$ = -a when a ≤ 0
$\quad\quad$ = +a when a ≥ 0

Calculating Roots

To find/simplify $\sqrt{n}$, prime factorize n and then group every pair of common factors and represent the factors with the number of groups as the exponent

Example:
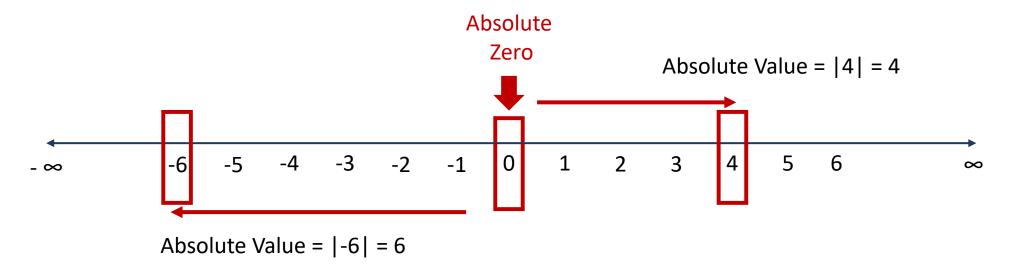
$$\sqrt{56} = \sqrt{2^3 \times 7^1} = \sqrt{2^2 \times 14} = 2\sqrt{14}$$

Therefore, $\sqrt{56} = 2\sqrt{14}$

Absolute Value

Definitions

Absolute Value is defined as the distance of any number from absolute zero.

Number Line:

Absolute
Zero

Absolute Value = |4| = 4

-6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6

- ∞   ∞

Absolute Value = |-6| = 6

Algebra

Equations

This is the form of a basic quadratic equation, $ax^2 + bx + c = 0$

And for all such equations, the general solution
for x can be found using the formula below:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Properties

$x^2 = a^2$

Solution: x = $\pm$ a

Square of any number will be always +ve (or 0 if the number itself is 0)

Equations with exponents:

$$x^a y^b = c z^d$$

where c is a constant

To solve this type of equation, first divide both sides by c, so that we have:

$$\frac{x^a y^b}{c} = z^d$$

Equations with Square Roots:

$$\sqrt{a} + b = c$$

where c is a constant

To solve this type of equation, Always take all the non-square root terms to one side of the equal to sign and the square root terms to the other side, then SQUARE BOTH SIDES (to remove the square root function):

$$\sqrt{a} + b = c$$

$$\sqrt{a} = c - b$$

$$(\sqrt{a})^2 = (c - b)^2$$

$$a = c^2 + b^2 - 2bc$$

Formula:

$(a + b)^2 = a^2 + b^2 + 2ab$

$(a - b)^2 = a^2 + b^2 - 2ab$

$a^2 - b^2 = (a + b)(a - b)$

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

$(a + b)^3 = a^3 + b^3 + 3ab(a + b)$

$(a - b)^3 = a^3 - b^3 - 3ab(a - b)$

Fractions

Definitions

Fractions are numerical representation of parts of a whole number

$$\frac{a}{b} = \frac{Numerator}{Denominator}$$

Numerator = The part that we want to represent
Denominator = The whole number that encompasses the part

Types of Fractions

Proper Fractions:

Fractions whose value falls between 0 & 1

Example:

$$\frac{2}{5}, \frac{3}{7}, \frac{8}{9}$$

Improper or Mixed Fractions:

Fractions whose value is greater than 1 or whose value is less than -1

Example:

$$2\frac{3}{8}, 6\frac{4}{9}, 1\frac{11}{34}$$

Properties of Fractions

Addition:

$$\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$$

$$\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$$

Subtraction:

$$\frac{a}{b} - \frac{c}{b} = \frac{a-c}{b}$$

$$\frac{a}{b} - \frac{c}{d} = \frac{ad-bc}{bd}$$

Multiplication/Division:

$$\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$$

$$\frac{a/b}{c/d} = \frac{ad}{bc}$$

Reciprocal:
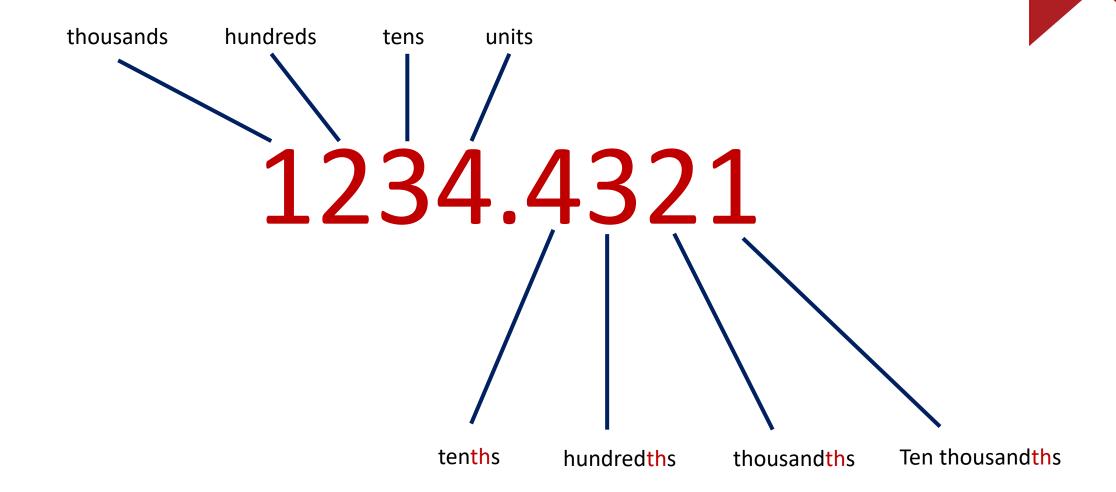
$$\left[\frac{a}{b}\right]^{-1} = \frac{b}{a}$$
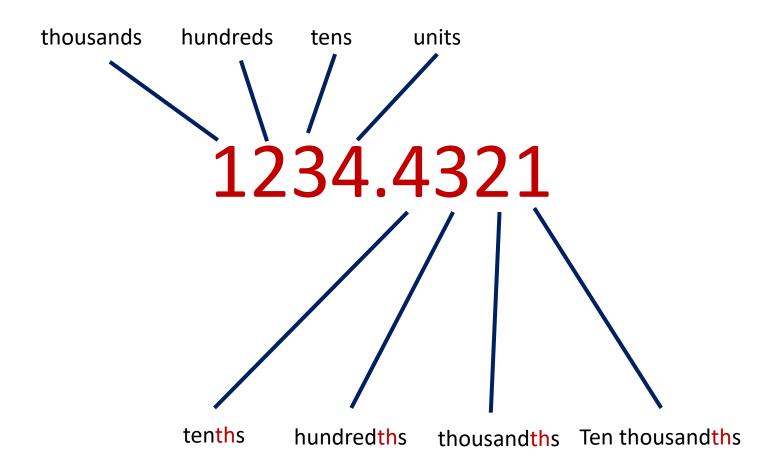
Additive Properties:

$$\frac{a}{b} < \frac{a+n}{b+n} \qquad \text{if } 0 < \frac{a}{b} < 1$$

$$\frac{a}{b} > \frac{a+n}{b+n} \qquad \text{if } \frac{a}{b} > 1$$

Digits & Decimals

Definitions

thousands   hundreds   tens   units

1234.4321

tenths   hundredths   thousandths   Ten thousandths

Definitions

thousands   hundreds   tens   units

1234.4321

tenths   hundredths   thousandths   Ten thousandths

It is important to note the difference between the names of the places of the digits.

All digit place names AFTER the decimal point have a th towards the end of their names

Rounding Off

Rounding up or rounding down is a very common function used in mathematics.
It is used as an approximation in order to simplify calculations and/or estimate values for other purposes.
It is also very commonly used to determine a range or fixed point intervals.

Rounding Up:

If the concerned digit is greater than 5, we round up

Example:

47 becomes 50
This is because 7>5
Keep in mind that 47 has been rounded to the closest tens place

Rounding Down:

If the concerned digit is lesser than 5, we round down

Example:

982 becomes 980
This is because 2<5
Keep in mind that 982 has been rounded to the closest tens place

Properties

Converting a recurring decimal into a Fraction:

$$= \frac{\textit{Recurring Number}}{\textit{equal number of 9s as repeating digits}}$$

Example:

$$0.41414141 = 0.\overline{41} = \frac{41}{99}$$

Percentages

Definitions

A percentage is a representation of a ratio as a fraction of 100 parts

If x is y % of z then,

$$x = \frac{y}{100} \times z$$

Example:

What percentage of 200 is 75?

Replacing the values in the above equation, we get,

$$75 = \frac{y}{100} \times 200$$

$$\Rightarrow y = 37.5\%$$

Example:

What is 45% of 450?

$$x = \frac{45}{100} \times 450$$

$$\Rightarrow x = 202.5$$

Formula

Percentage Increase:

$$\% \uparrow = \frac{Final\ Value - Original\ Value}{Original\ Value} \times 100$$

Example:

Find the % increase between 550 & 650

$$= \frac{Final\ Value - Original\ Value}{Original\ Value} \times 100$$

$$= \frac{650 - 550}{550} \times 100 = 18.18\% \text{ (increase)}$$

Percentage Decrease:

$$\% \downarrow = \frac{Original\ Value - Final\ Value}{Original\ Value} \times 100$$

Example:

Find the % decrease between 980 & 2000

$$= \frac{Final\ Value - Original\ Value}{Original\ Value} \times 100$$

$$= \frac{2000 - 980}{2000} \times 100 = 51\% \text{ (decrease)}$$

Formula

Simple Interest:

Compound Interest:

$$SI = \frac{\text{PxRxT}}{100}$$

$$CI = P(1 + \frac{R}{n \times 100})^{nT}$$

Where, P – Principal

R – Rate

T – Time

n – Number of times per year

Word Problems

Definitions

Addition

- Increased by
- More than
- Combined
- Together
- Total of
- Sum
- Added to

Definitions

Subtraction

- Decreased by
- Less
- Less than
- Fewer than
- Minus
- Difference between
- Difference of

Definitions

Multiplication

- Of
- Times
- Multiplied by
- Product of
- Increased by a factor of
- Decreased by a factor of

Definitions

Division

- Per
- Quotient of
- Ratio of
- Percent (%)

Definitions

Equals

- Is
- Are
- Was
- Were
- Will be
- Gives
- Yields
- Sold for

Properties

Sum of two consecutive Integers

n + (n+1)

Sum of two consecutive EVEN Integers

2n + (2n+2)

Sum of three consecutive EVEN Integers

(2n-2) + 2n + (2n+2)

Properties

Sum of two consecutive ODD Integers

(2n-1) + (2n+1)

Sum of four consecutive ODD Integers

(2n-3) + (2n-1) + (2n+1) + (2n+3)

Properties

x is 10% more than y

$$x = \left(1 + \frac{10}{100} \right) y = 1.1y$$

x is 10% less than y

$$x = \left(1 - \frac{10}{100} \right) y = 0.9y$$

Speed, Distance & Time

Definitions

Rate x Time = Distance

Or

Speed x Time = Distance

Or

$$\text{Time} = \frac{\textcolor{red}{Distance}}{Speed}$$

Average Speed

Average Speed = $\dfrac{\text{Total Distance Travelled}}{\text{Total Time Taken}}$

From A to B: Speed = v_1
From B to A: Speed = v_2

Average Speed = $\dfrac{d+d}{\dfrac{d}{v_1}+\dfrac{d}{v_2}}$ =

Speed Upstream

Speed of Boat (in stagnant water) = v_1
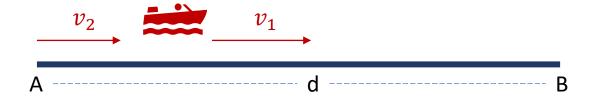
Speed of Stream = v_2

Speed of Boat Upstream = v_1 - v_2

Speed Downstream

Speed of Boat (in stagnant water) = v_1

Speed of Stream = v_2

Speed of Boat Downstream = v_1 + v_2

Two Approaching Trains
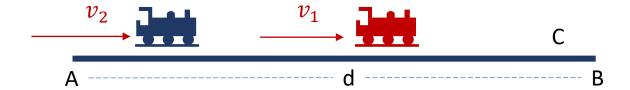
Two approaching trains A and B, travelling at a speed of v_1 and v_2 respectively, will meet at time t given by:

$$t = \frac{d}{v_1 + v_2}$$

$$\text{Distance AC} = = \frac{v_1 d}{v_1 + v_2} \qquad \text{Distance BC} = = \frac{v_2 d}{v_1 + v_2}$$

Two Parallel Trains

Two approaching trains A and B, travelling at a speed of v_1 and v_2 respectively, will meet at time t given by:

$$t = \frac{d}{v_2 - v_1}$$

Rate & Work

Definitions

Rate x Time = Work

If the Work done is the same throughout the question then Work is usually considered to be 1 (unit)

If work done is in parts – 20 Walls painted & 25 Walls painted, then the work(s) being done are 20 & 25 respectively

Definitions

Rate x Time = Work

If two people are working together then their combined Rate of Work,

$$R = R_1 + R_2$$

Where R_1 is the rate of person 1 working alone
&
R_2 is the rate of the person 2 working alone

Definitions

Rate x Time = Work

If there are 10 people working together with each working with the rate of R_1, then combined Rate of Work,

$$R = 10R_1$$

Definitions

Calculating the Time Taken to finish the Work
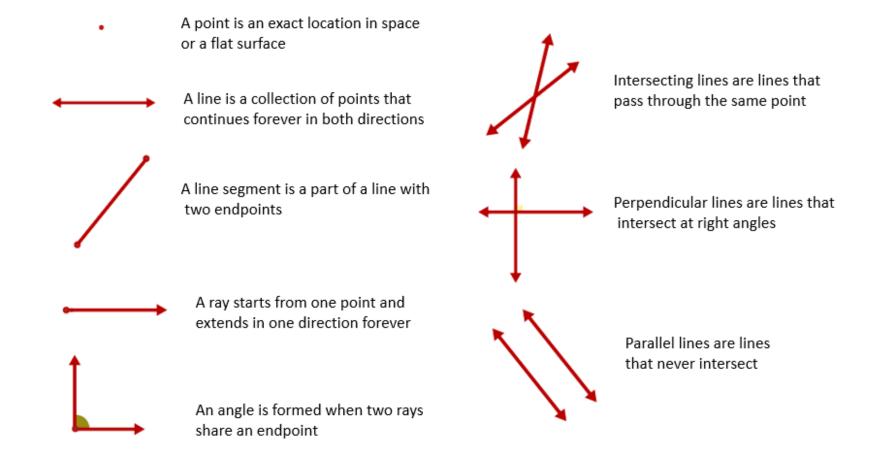
$$\frac{1}{\text{Time taken by }A \text{ to finish the Job}} + \frac{1}{\text{Time taken by }B \text{ to finish the Job}} = \frac{1}{\text{Time taken by }A \text{ and }B \text{ together to finish the Job}}$$
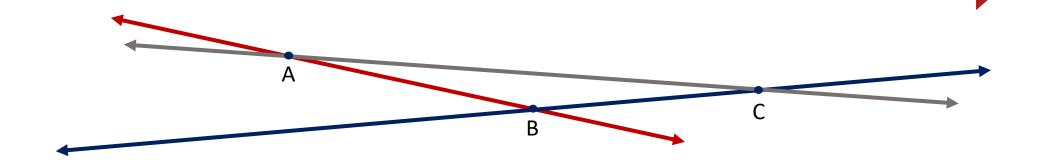
Geometry

Definitions

A point is an exact location in space or a flat surface

A line is a collection of points that continues forever in both directions

A line segment is a part of a line with two endpoints

A ray starts from one point and extends in one direction forever

An angle is formed when two rays share an endpoint

Intersecting lines are lines that pass through the same point

Perpendicular lines are lines that intersect at right angles

Parallel lines are lines that never intersect

Definitions

When 2 or more lines meet at a certain point, that point is called the intersection of those lines

At these intersections, the lines form angles
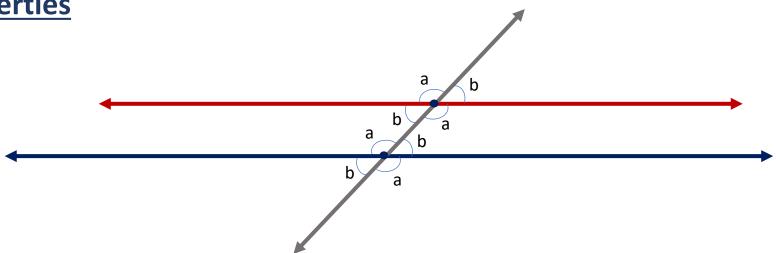
There are primarily 6 different types of angles

An Angle is defined as a figure which has 2 rays emerging from a single common point

By this definition of angles, we can see that multiple angles are formed when two (or more) lines intersect

Properties

When 2 or more lines meet at a certain point, that point is called the intersection of those lines
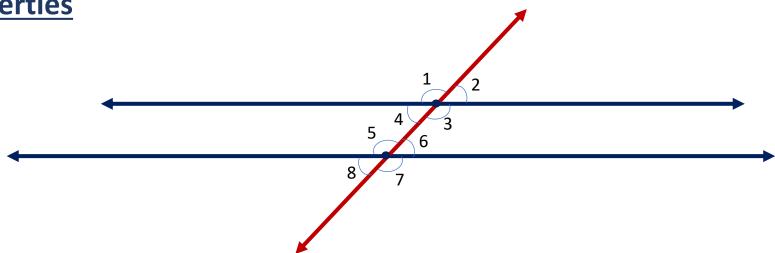
At these intersections, the lines form angles. There are primarily 6 different types of angles

Angles are defined as a figure which has 2 rays emerging from a single common point

By this definition of angles, we can see that multiple angles are formed when two (or more) lines intersect

$\angle a + \angle b + \angle a + \angle b = 360°$ => $\angle a + \angle b = 180°$ [for all $\angle a$ & $\angle b$]

Properties

This diagram can also be labelled in a different way to explain the properties of angles formed by parallel lines and a transversal

$\angle1 + \angle2 + \angle3 + \angle4 = 360° \Rightarrow \angle1 + \angle2 = 180° \Rightarrow \angle2 + \angle3 = 180° \Rightarrow \angle3 + \angle4 = 180° \Rightarrow \angle4 + \angle1 = 180°$
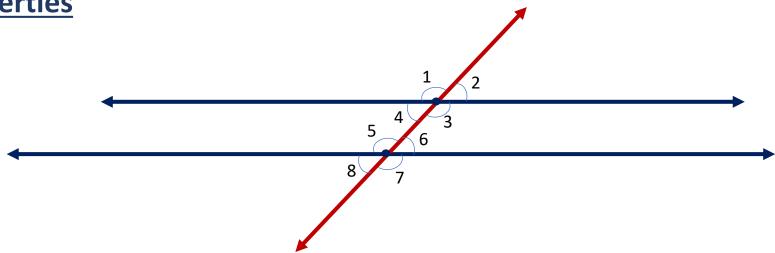
$\angle5 + \angle6 + \angle7 + \angle8 = 360° \Rightarrow \angle5 + \angle6 = 180° \Rightarrow \angle6 + \angle7 = 180° \Rightarrow \angle7 + \angle8 = 180° \Rightarrow \angle8 + \angle5 = 180°$

From this we can infer that

$$\angle1 = \angle3 = \angle5 = \angle7$$
AND
$$\angle2 = \angle4 = \angle6 = \angle8$$

Properties

Vertically Opposite Angles	Corresponding Angles	Alternate Interior Angles	Alternate Exterior Angles
$\angle 1 = \angle 3$	$\angle 1 = \angle 5$	$\angle 4 = \angle 6$	$\angle 1 = \angle 7$
$\angle 5 = \angle 7$	$\angle 3 = \angle 7$	$\angle 3 = \angle 5$	$\angle 2 = \angle 8$
$\angle 2 = \angle 4$	$\angle 2 = \angle 6$		
$\angle 6 = \angle 8$	$\angle 4 = \angle 8$		
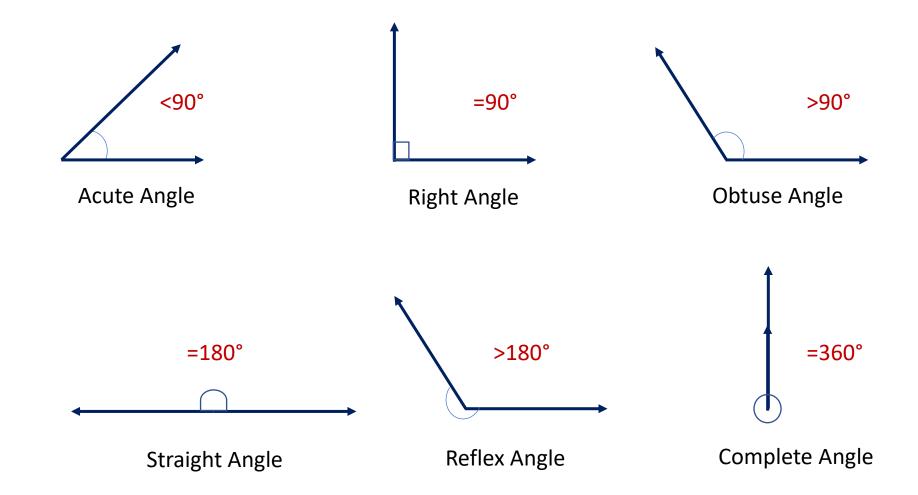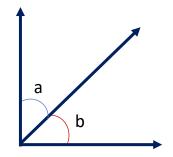
Types of Angles

<90°

Acute Angle

=90°

Right Angle

>90°

Obtuse Angle

=180°

Straight Angle

>180°

Reflex Angle

=360°

Complete Angle
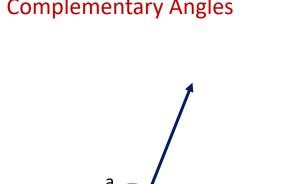
Types of Angles

Complementary Angles

$\angle$a + $\angle$b = 90°

When the sum of 2 angles equals 90°, they are called Complementary angles

Supplementary Angles

$\angle$a + $\angle$b = 180°

When the sum of 2 angles equals 180° , they are called Supplementary angles

Triangles

Definitions
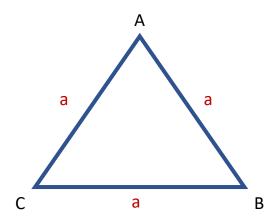
A triangle has **3 sides** and **3 interior angles**

The **3 angles** always sum up to **180°**

Area of a **Triangle** = $\frac{1}{2}$ x **Base** x **Height**

= $\frac{1}{2}$ x **CB** x **AD**

Types of Triangles
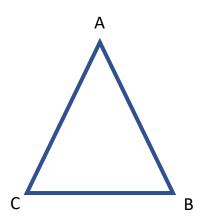
Equilateral Triangle

3 EQUAL sides

3 EQUAL interior angles

Area of a Triangle = $\frac{\sqrt{3}}{4}$ x a^2
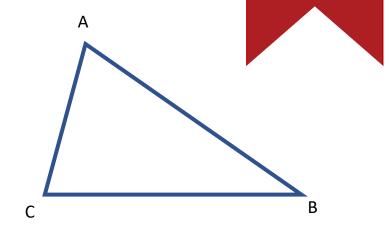
Isosceles Triangle

2 EQUAL sides

2 EQUAL interior angles

Area of a Triangle = $\frac{1}{2}$ x b x h
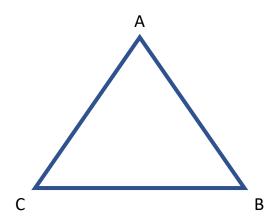
Scalene Triangle

NO EQUAL sides

NO EQUAL interior angles

Area of a Triangle = $\frac{1}{2}$ x b x h

Types of Triangles
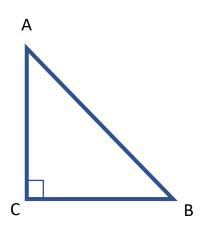
Acute Angled Triangle

All angles are less than 90°

Sum of all angles = 180°

Area of a Triangle = $\frac{1}{2}$ x b x h
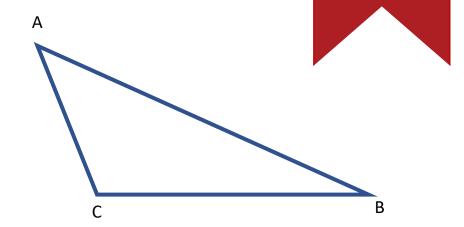
Right Angled Triangle

Exactly one angle = 90°

Sum of all angles = 180°

Area of a Triangle = $\frac{1}{2}$ x b x h
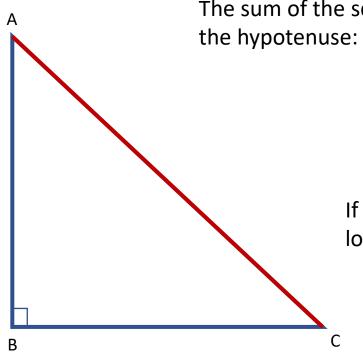
Obtuse Angled Triangle

Exactly one angle > 90°

Sum of all angles = 180°
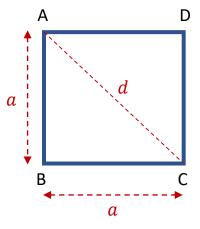
Area of a Triangle = $\frac{1}{2}$ x b x h

Pythagoras Theorem

The sum of the squares of two sides of a right angled triangle is equal to the square of the hypotenuse:

$$AB^2 + BC^2 = AC^2$$

If we extended △ABC into a square with the same dimensions, it would look like this:
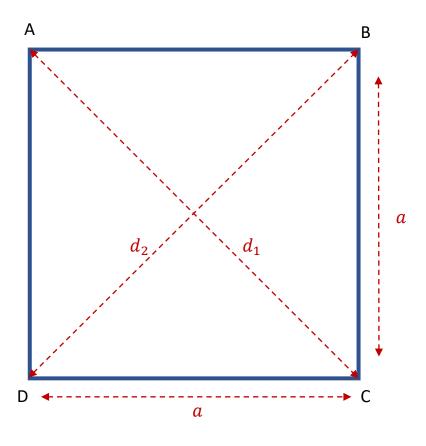
And we know that the length of the diagonal of a square, d = $\sqrt{2}a$

This concept is what the Pythagoras Theorem is based on

Quadrilaterals

Square

A ─────────── B

d_2 d_1

a

D ─────────── C

a
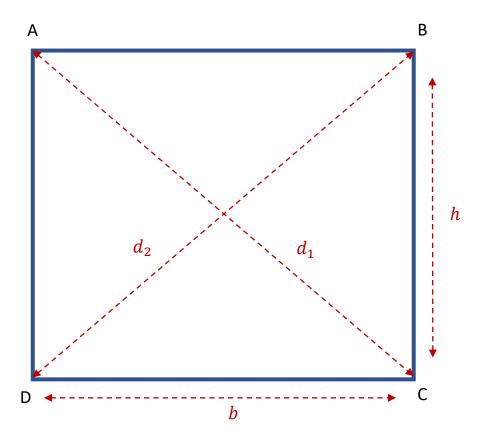
- Figure ABCD is a square with sides = a and diagonals d_1 & d_2

- All 4 sides are equal
- Opposite sides are equal and parallel
- All angles are 90°
- Sum of all angles = 360°
- Diagonals are equal and perpendicularly bisect each other

- Area of Square = a^2
- Perimeter of a Square = $4a$
- Diagonal, d = $\sqrt{2}a$
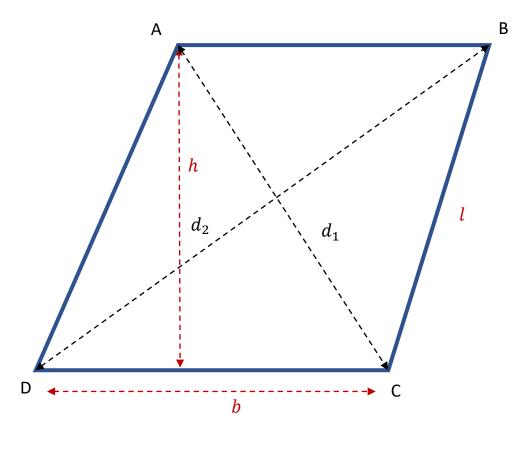
Rectangle

- Figure ABCD is a rectangle with sides = b, h
  and diagonals d_1 & d_2

- Only opposite sides are equal and parallel
- All angles are 90°
- Sum of all angles = 360°
- Diagonals are equal and perpendicularly bisect each other

- Area of Rectangle = b x h
- Perimeter of a Rectangle = $2(b + h)$
- Diagonal, d = $\sqrt{(b^2 + h^2)}$
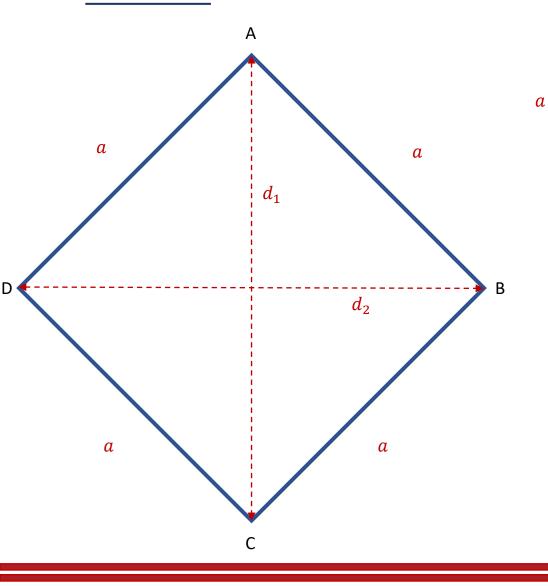
Parallelogram

- Figure ABCD is a parallelogram with sides = l, b and diagonals d_1 & d_2

- Only opposite sides are equal AND parallel
- Opposite angles are equal
- Sum of all angles = 360°
- Diagonals are equal and perpendicularly bisect each other

- Area of Parallelogram = b x h
- Perimeter of a Parallelogram = $2(b + l)$
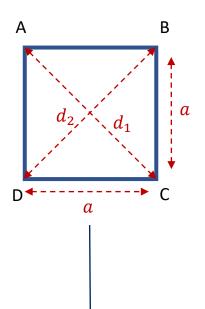
Rhombus
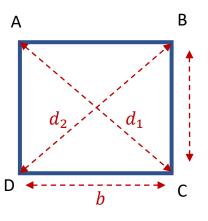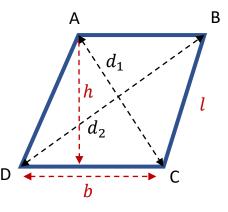
- Figure ABCD is a rhombus with sides = a and diagonals d_1 & d_2

- Opposite sides are equal AND parallel
- Opposite angles are equal
- None of the angles are 90°
- Sum of all angles = 360°
- Diagonals are equal and perpendicularly bisect each other

- Area of Rhombus = $\frac{1}{2}$ x d_1 x d_2
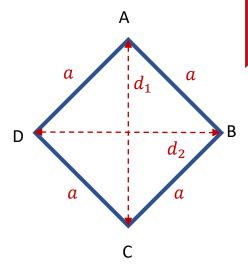- Perimeter of a Rhombus = $\sum a$

Formula

- Area of Square = a^2
- Perimeter of a Square = $4a$
- Diagonal, d = $\sqrt{2}a$
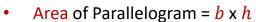
- Area of Rectangle = b x h
- Perimeter of a Rectangle = $2(b + h)$
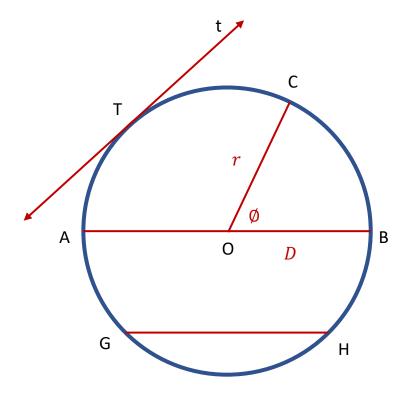- Diagonal, d = $\sqrt{(b^2 + h^2)}$

- Area of Parallelogram = b x h
- Perimeter of a Parallelogram = $2(b + l)$

- Area of Rhombus = $\frac{1}{2}$ x d_1 x d_2
- Perimeter of a Rhombus = $\sum a$

Circles

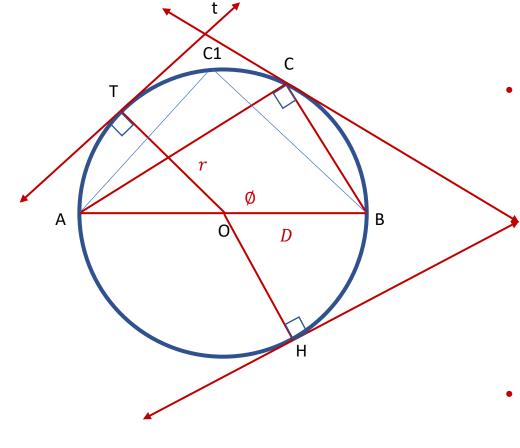# Circle

- Area of a circle $= \pi r^2$
- Circumference (perimeter) of circle = $2\pi r$

- AB is the diameter, D
- OC is the radius, r
- r = $\frac{D}{2}$

- GH is a chord

- t is the tangent to the circle meeting it at point T

- CB is an Arc. So is ATC
- Length of Arc CB = $\frac{\emptyset}{360}$ x $2\pi r$
- COB is a sector. Area of sector = $\frac{\emptyset}{360}$ x πr^2
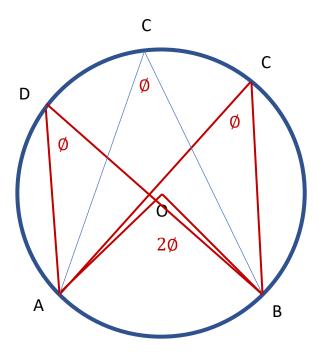
Circle

- Diameter subtends a right angle anywhere on the circle ∠ACB = 90

- Length of all tangents (only two are possible) drawn to a circle from a single point are equal
PH=PC

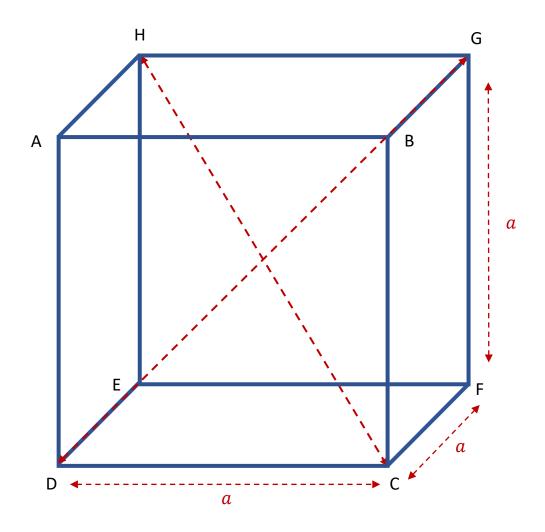- Angle formed by radius drawn to the point of contact of tangent is always 90
PHO = 90   OTA = 90

Circle

- An Arc subtends the same angle at any point on the circle ∠ACB = ∠ADB = Ø

- Angle subtended by an arc at the centre is twice the angle subtended by the arc at any point on the circle
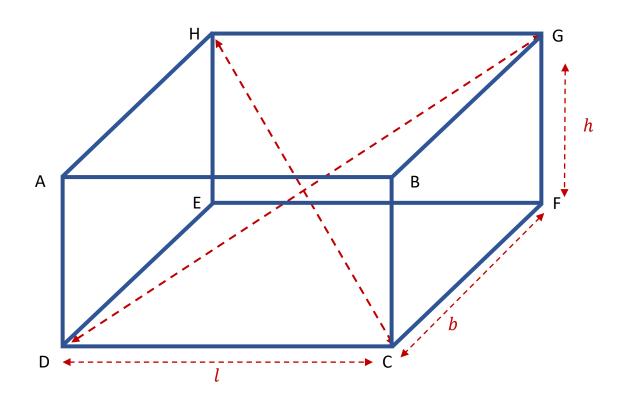
$$∠AOB = 2 \times ∠ACB = 2 \times ∠ADB = 2 \times Ø$$
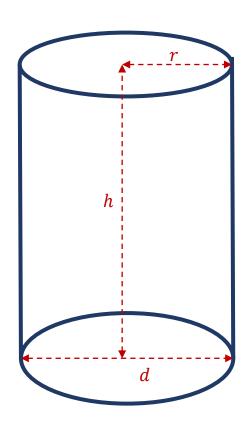
3D Geometry

Cube

- Volume of Cube = a^3
- Surface Area of a Cube = $6a^2$
- Longest Diagonal, d = $\sqrt{3}a$

- All edges are equal in length
- All faces are equal in area
- 6 Faces
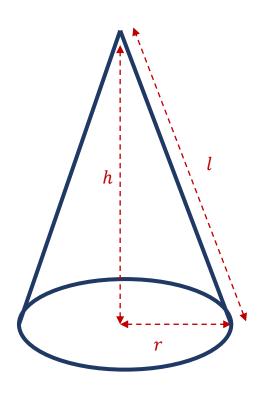- 12 Edges
- HC & GD are the longest diagonals

Cuboid

- **Volume** of Cuboid = l x b x h
- **Surface Area** of a Cuboid = $2(lb + bh + hl)$
- **Longest Diagonal**, d = $\sqrt{(l^2 + b^2 + h^2)}$

- Opposite edges are equal in length
- Opposite faces are equal in area
- 6 Faces
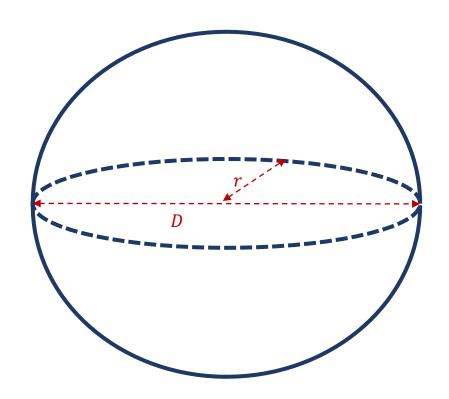- 12 Edges
- HC & GD are the longest diagonals

Cylinder

- Volume of Cylinder = $\pi r^2 h$
- Surface Area of a Solid Cylinder = $2\pi r^2 h$ + $2\pi rh$

- Surface Area of a Hollow Cylinder = $2\pi rh$

Cone
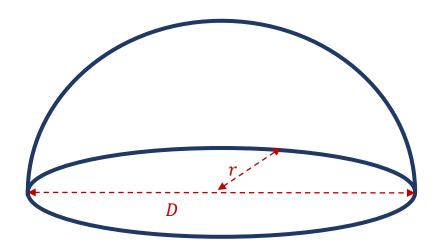
- Volume of Cone = $\frac{1}{3}\pi r^2 h$
- Surface Area of a Cone = $\pi r^2 + \pi r l$

- Length of a Cone = $\sqrt{(r^2 + h^2)}$

Sphere

- **Volume** of Sphere = $\frac{4}{3}\pi r^3$
- **Surface Area** of a Sphere = $4\pi r^2$
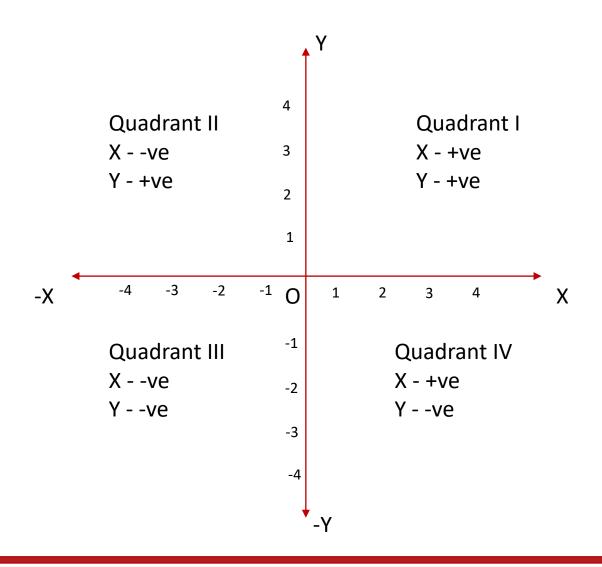
Hemisphere
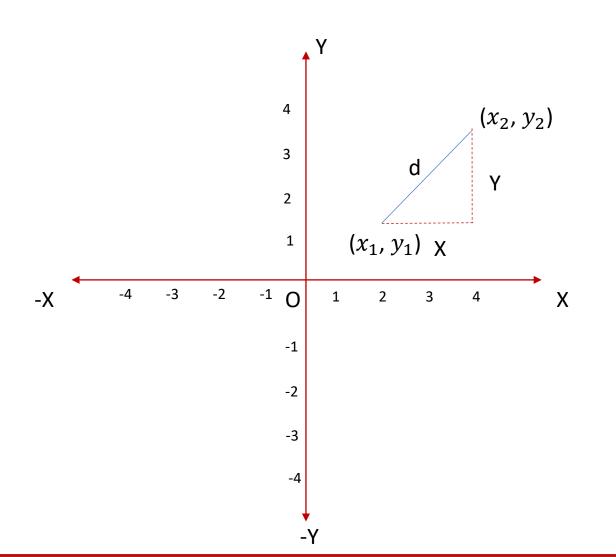
- Volume of Hemisphere = $\frac{2}{3}\pi r^3$
- Surface Area of a Solid Hemisphere = $3\pi r^2$
- Surface Area of a Hollow Hemisphere = $2\pi r^2$

Coordinate Geometry

Coordinate Geometry

Quadrant II
X - -ve
Y - +ve

Quadrant I
X - +ve
Y - +ve

Quadrant III
X - -ve
Y - -ve

Quadrant IV
X - +ve
Y - -ve

Distance between two points
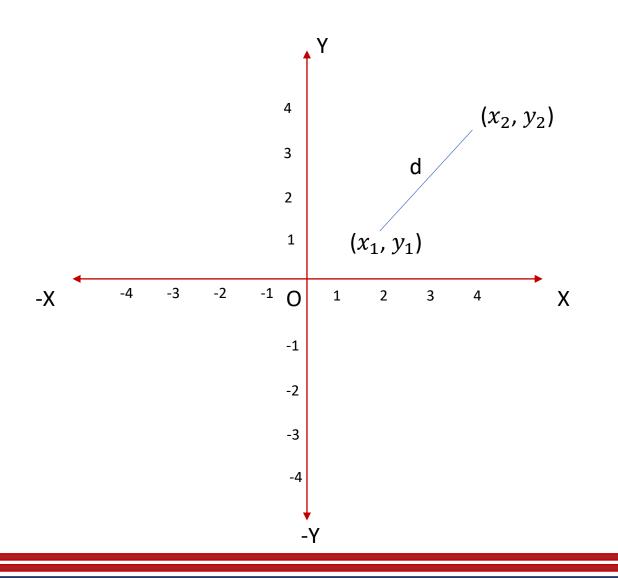
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\text{Slope, m} = \frac{(y_2 - y_1)}{(x_2 - x_1)}$$

Equation of a line on the coordinate plane:

$$y = mx + c$$

Distance between two points
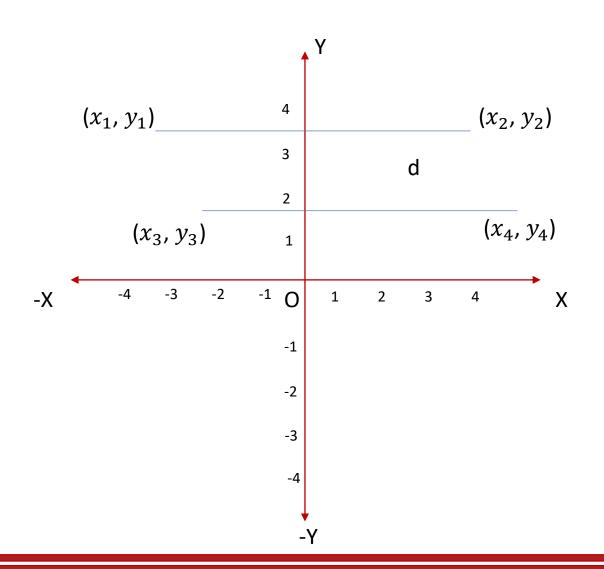
Equation of a line passing through two points:

$$\frac{(y-y_1)}{(x-x_1)} = \frac{(y_2-y_1)}{(x_2-x_1)}$$

Equation of a line on the coordinate plane:

$$y = mx + c$$

Distance between parallel lines

(x_1, y_1) _____ (x_2, y_2)

d

(x_3, y_3) _____ (x_4, y_4)

Y

4

3

2

1

-4   -3   -2   -1   O   1   2   3   4

-X   X

-1

-2

-3

-4

-Y
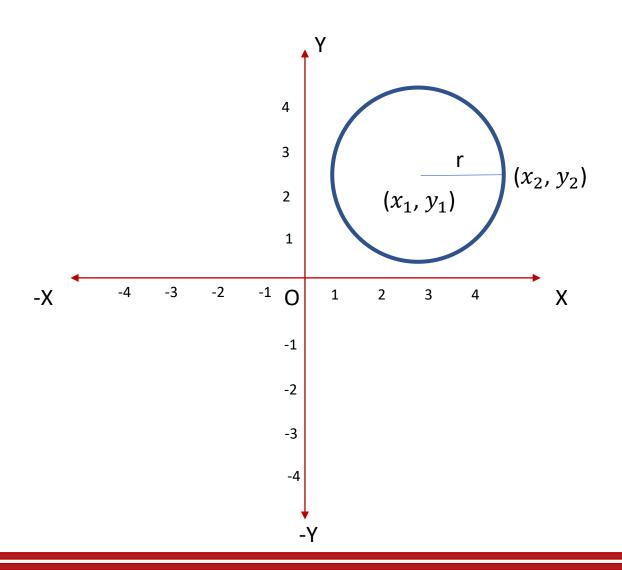
Equation of a line:

$$ax + by + c_1 = 0$$
$$ax + by + c_2 = 0$$

Distance of 2 parallel lines on the coordinate plane:

$$d = \frac{|c_1 - c_2|}{\sqrt{a^2 + b^2}}$$

Equation of other figures

4

3

r

(x_2, y_2)

2

(x_1, y_1)

1

-X   -4   -3   -2   -1 O   1   2   3   4   X

-1

-2

-3

-4

-Y

Equation of a circle with centre at (x_1, y_1) :

$$(x - x_1)^2 + (y - y_1)^2 = r^2$$

Statistics

Definitions

Mean

Average of all the numbers in the set

For instance, if a set has 5 elements a_1, a_2, a_3, a_4, a_5 , then the sum is given by:

$$\text{Sum} = a_1 + a_2 + a_3 + a_4 + a_5$$

Therefore, the Mean or Average = $\dfrac{a_1 + a_2 + a_3 + a_4 + a_5}{5}$

Definitions

Median

The middle value of a set

For instance, if a set A has { 1,1,2,3,4,6}

The Median is $\frac{2+3}{2}$ = 2.5

In case of an odd number of elements in a set, the middle element of the set is the median

Definitions

Mode

The number that occurs the greatest number of times within a set

For instance, if a set A has { 1,1,1,2,2,2,3,3,3,3,3,3,3,3,3,3,3,3,4,4,}

The Mode is 3 as 3 has occurred the maximum number of times compared to the other elements in the set A

Definitions

Range

The difference between the highest and lowest value of the set
For instance, if a set A has {1,2,3,4,5,6}

The Range is 6 - 1 = 5

Weighted Average

$$\text{Weighted Average} = \frac{(Weight\,.Value)+\cdots+(Weight\,.Value)}{Sum\,of\,Weights}$$

Definitions

Weighted Average

Example:

    If an employee's performance review consists of

        20% Component A

        30% Component B

        50% Component C
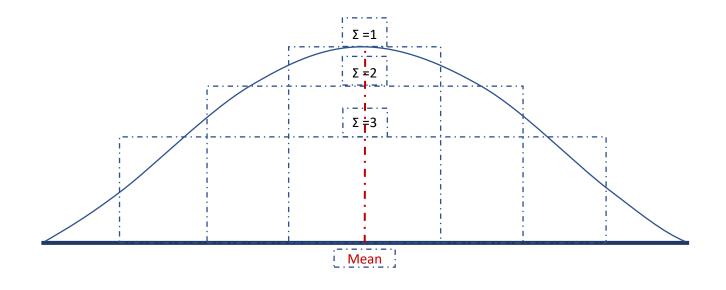
And he receives 10 points in A, 20 in B and 10 in C, then his overall performance is given by

$$\text{Weighted Average} = \frac{(0.2 \cdot 10) + (0.3 \cdot 20) + (0.5 \cdot 10)}{0.2 + 0.3 + 0.5}$$

Definitions

Standard Deviation

It is defined as the indicator of how the numbers of a range are spread.
It is equal to the Root Mean Square [RMS] of the distance of the values from the Mean.

Definitions

Variance

$$\text{Variance} = \frac{\textcolor{red}{Sum~of~the~squares}~of~\textcolor{red}{the~difference}~of~\textcolor{red}{each~number}~from~the~\textcolor{red}{mean}}{Total~number~of~numbers}$$

Standard Deviation

$$\text{Standard Deviation} = \sqrt{Variance}$$

Definitions

Example

Set is {1,2,3,4,5}

$$\text{Mean} = \frac{1+2+3+4+5}{5} = 3$$

$$\text{Variance or V} = \frac{(3-1)^2 + (3-2)^2 + (3-3)^2 + (3-4)^2 + (3-5)^2}{5} = \frac{4+1+0+1+4}{5} = 2$$

Standard Deviation = $\sqrt{Variance} \Rightarrow \sqrt{2}$

Therefore, Standard Deviation = $\sqrt{2}$

Probability

Definitions

Probability

Probability or likelihood is a measure or estimation of how likely it is that an event will happen
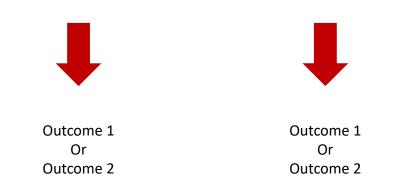
Formula

$$\text{Probability} = \frac{Number\ of\ successful\ outcomes}{Total\ number\ of\ possible\ outcomes}$$

Definitions
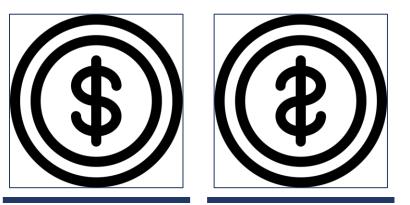
Probability is a simple concept if you can try and understand it from a logical point of view rather than trying to figure out what all the formulae mean

What do you understand if someone were to say that a fair coin is tossed once?

It is obvious that every coin has only two sides. Therefore we can have one of two possible outcomes. It can be either Heads OR Tails.

So there are 2 possible outcomes, however the probability of either outcome is $\frac{1}{2}$

Outcome 1
Or
Outcome 2

Outcome 1
Or
Outcome 2
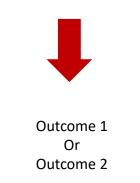
Here, on the first toss, it can ONLY be 1 of 2 outcomes – Heads OR Tails

It can be either Heads OR Tails. But what is the likelihood that it is Heads?

$$\frac{1}{2}$$

— *Number of successful outcomes*

— You toss the coin and you have two options

Outcome 1
Or
Outcome 2

Outcome 1
Or
Outcome 2

What do you understand if someone were to say that a fair coin is tossed TWICE?

In the previous scenario, we tossed the coin only once, but if we toss it twice, what becomes the new probability or likelihood of getting HEADS on BOTH tries?

Permutation

Definitions

These follow the fundamental principles of counting.

If an operation can be performed in m different ways & another operation can be performed in n different ways then

- The two operations can be performed one after the other in mn ways
- Either of these two operations can be performed in $m + n$ ways (provided only one has to be done)

Definitions

Arrangement of n different objects in a row can be done in n! ways

Arrangement (N) = 1 x 2 x 3 x …… x (n-1) x n

Permutation

- An arrangement of things in a given order
- Order Matrix – AB & BA are considered two separate arrangements
- Number of permutation of r objects out of n is given by

$$P(n,r) = {}^{n}_{r}P = \frac{n!}{(n-r)!}$$

Definitions

Number of permutation of n different things taking not more than r at a time:

$$^{n}P_{1} + {}^{n}P_{2} + {}^{n}P_{3} + {}^{n}P_{4} + ... + {}^{n}P_{r}$$

Definitions

Number of permutation of m different things taken r at a time, when a particular thing is to be always included in each arrangement:

$$^{m-1}_{r-1}P \text{ x } r$$

Definitions

Number of permutation of m different things taken r at a time, when a particular thing is NEVER to be included in each arrangement:

$$^{m-1}_rP$$

Definitions

Number of permutation of n different things taken all at a time, when m specified elements always come together:

$$m! \times (n - m + 1)!$$

Definitions

Number of permutation of m dissimilar things taken k at a time when r(where r<k) particular things always occur is:

$$^{m-r}_{k-r}P \times \, ^{k}_{r}P$$

Definitions

Number of permutation of m dissimilar things taken k at a time when r particular things never occur is:

$$^{m-r}_{k}P$$

Definitions

Number of permutation of m dissimilar things taken k at a time when repetition of things is allowed any number of times is:

$$m^k$$

Definitions

Number of permutation of k dissimilar things taken not more than r at a time, when each thing can occur:

$$k^1 + k^2 + k^3 + \ldots + k^r = \frac{k(k^r - 1)}{k - 1}$$

Definitions

Number of permutation of n different things taken all at a time, when m specified elements always come together:

$$n! - [m! \text{ x } (n - m + 1)!]$$

Definitions

Number of permutations of m different things taken all at a time, when k of them are all alike and the rest are different:

$$\frac{m!}{k!}$$

Circular Permutations

Definitions

Number of ways in which m different things can be arranged in a circle is:

$$\frac{n!}{n} = (n-1)!$$

Definitions

Number of circular permutations of m different things taken k at a time:

$$\frac{^m_k P}{k}$$

Definitions

Number of circular permutations of m different things taken k at a time in one direction:

$$\frac{^m_kP}{2k}$$

Definitions

Number of circular permutations of m different things in clockwise direction = Number of permutation in anticlockwise direction:

$$\frac{(m-1)!}{2}$$

Combination

Definitions

Number of combinations of m different things taken k at a time:

-> r particular things will ALWAYS occur:

$$_{k-r}^{m-r}C$$

Definitions

Number of combinations of m different things taken k at a time:

-> r particular things will NEVER occur:

$$_{k}^{m-r}C$$

Definitions

Number of combinations of m different things taken k at a time:

-> a particular things will ALWAYS occur AND b particular things NEVER occur:

$$_{k-a}^{m-b-a}C$$

Sets

Definitions

Condition 1 & Condition 2

Total

	Yes C1	No C1	Total
Yes C2			
No C2			
Total			

Definitions

Dogs and Cats:

100 houses in the neighbourhood

25 both cats and dogs

40 have only dogs

55 houses have cats

	Dogs	No dogs	Total
Cats	25		55
No Cats	40		
Total			100